# Tailoring calibration

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# How do we know what to do?

- ➩ How do you know what parameters to set?
  - ☞ e.g what solution interval for phase calibration?
- ➩ Some…

  depend on physical/instrument properties (fixed for a given observation)
- ➩ Others…

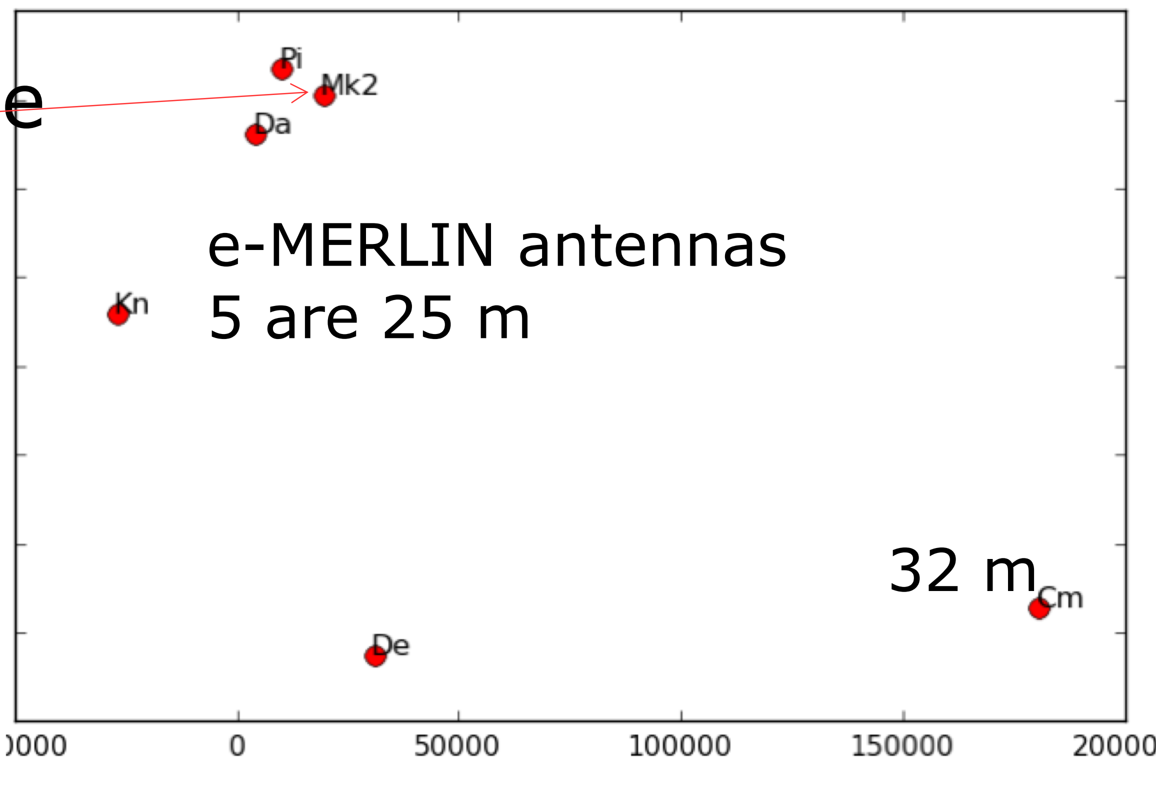
  depend on your science goals
- ➩ Image pixel size: >3 pixels across synthesised beam ($\theta$ = min_$\lambda$/max_baseline (c.f. Imaging talk)
- ➩ Easy to pipeline

# Observation-dependent parameters

- ➯ Some things Calibration strategy depends on…
  - ☞ Observing frequency, baselines etc.
  - ☞ Weather and source elevation
  - ☞ Calibration source properties
- ➯ Imaging depends on all these and on science goals
  - ☞ Faint, extended source?
  - ☞ Very bright, self-calibratable source?
  - ☞ Spectral lines?
  - ☞ Sources all over the field of view?

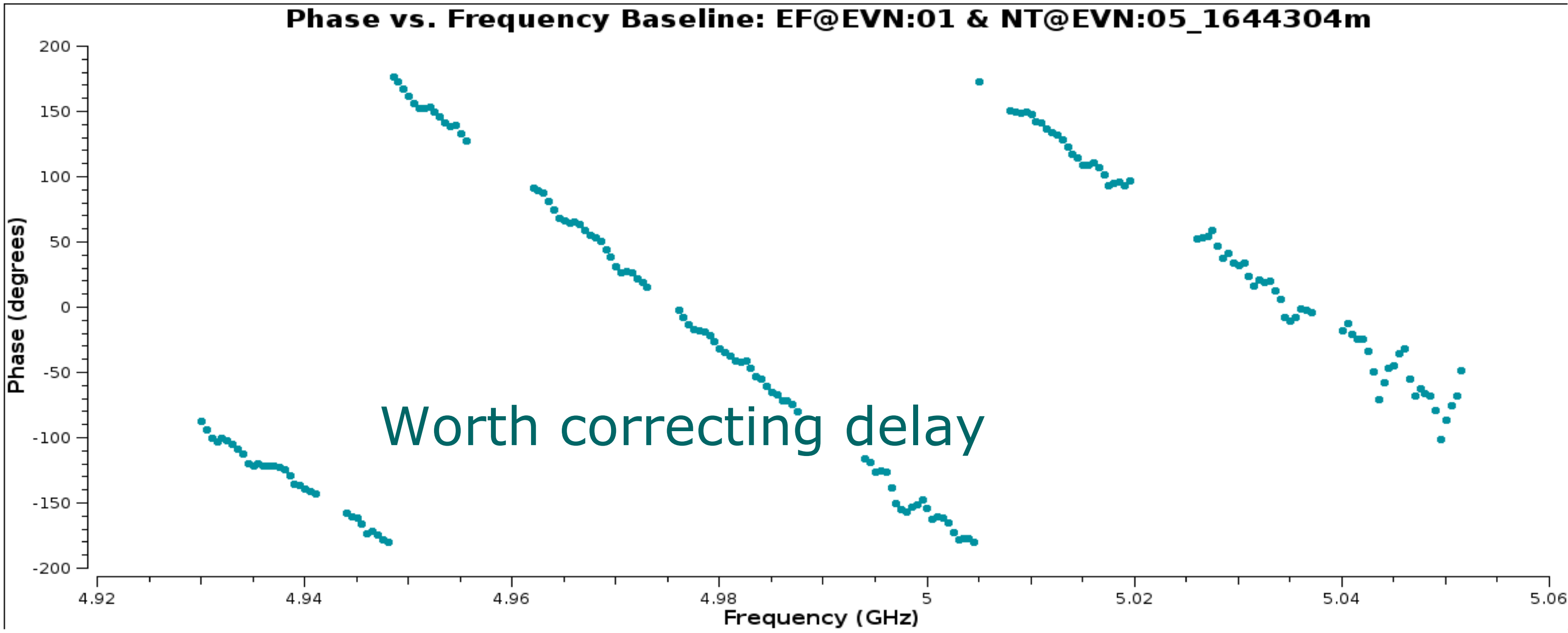
# Choosing reference antenna

- ➾ The antenna with the best chance of good solutions to all other antennas
  - ☞ Most short baselines? Greater atmospheric differences on long baselines
  - ☞ Most sensitive?
- ➾ e-MERLIN: usually use Mk2 (or Pi or Da)
- ➾ EVN: Ef >> most sensitive and central



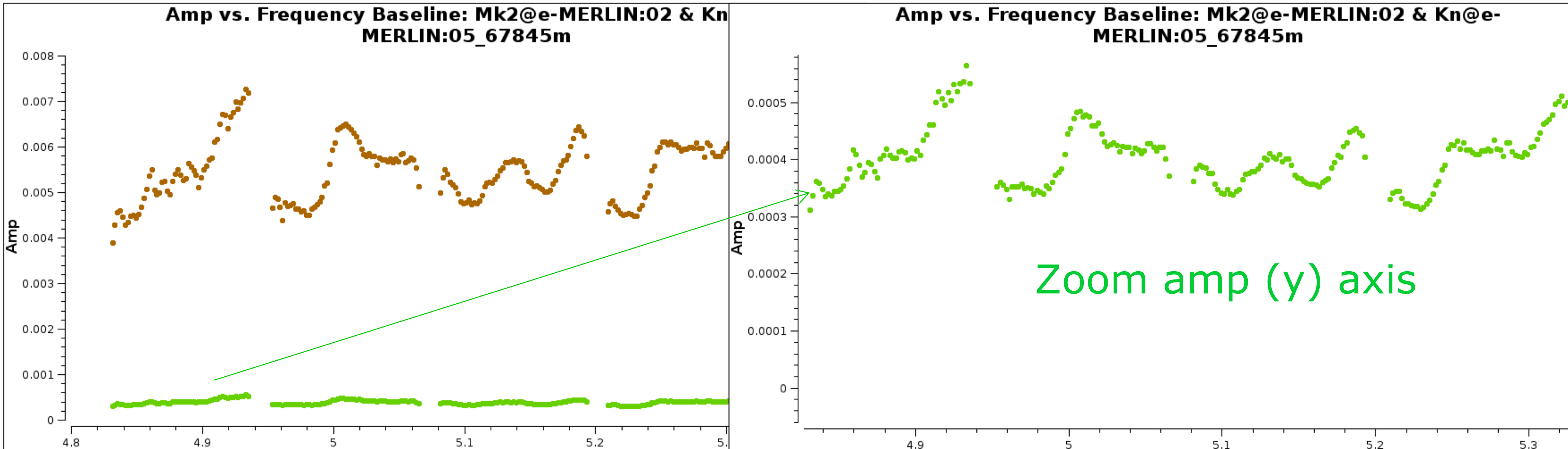
# Delay calibration

- Delay corrections for linear phase gradients:
- Inspect phase v. frequency
- Only worth correcting delay if you can see it
- Usually stable for hours but averaging solint limited (~scan) by time-dependent phase stability



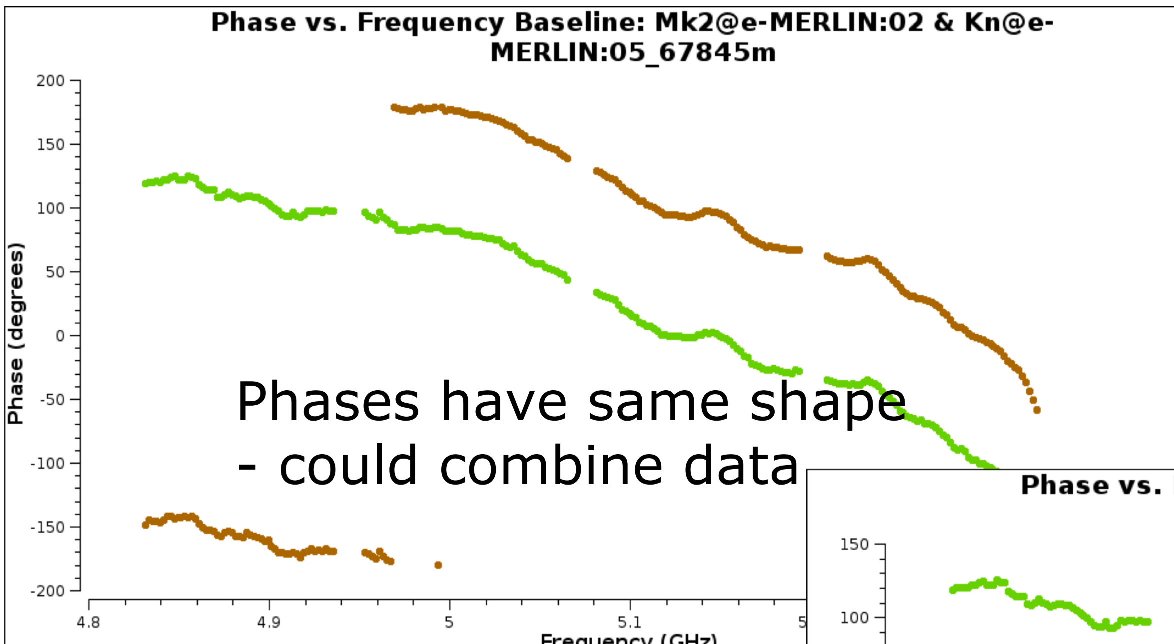
# Bandpass calibration

- ⇨ Correct BP cal phase v. time first (see following slides)
- ⇨ In Bandpass, average in time for as long as possible for best S/N per channel
  - ⇨ Both BP cals have same amp wiggles
  - ⇨ Could combine, interpolate or use just the one with best S/N

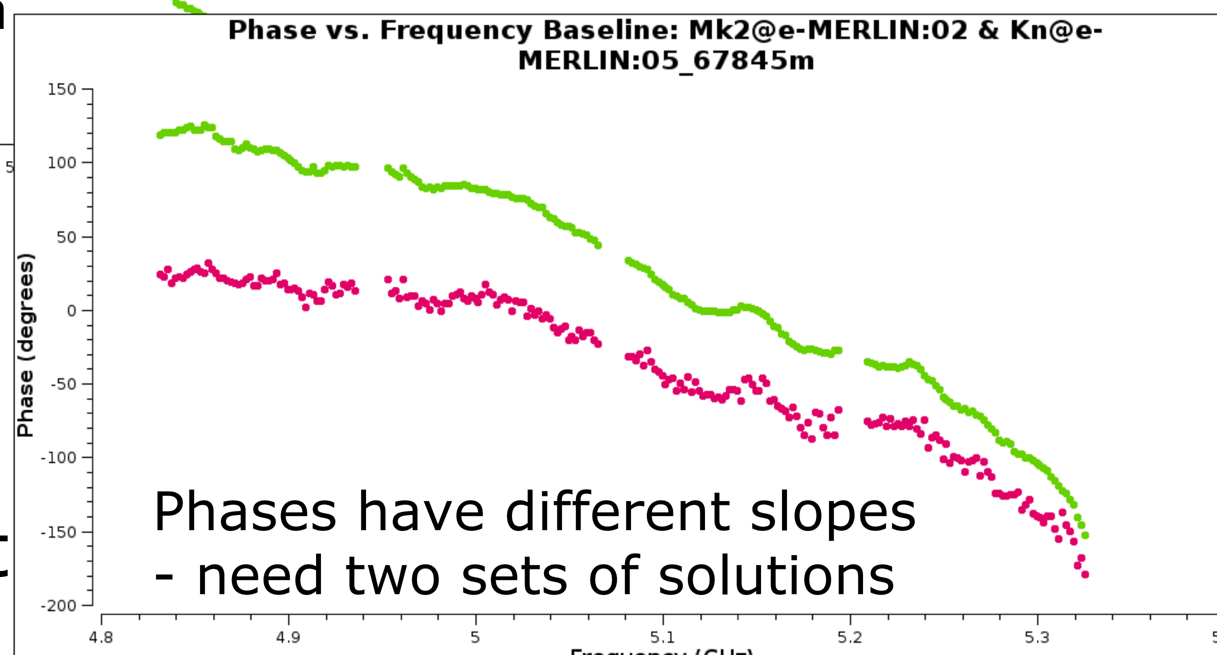


# Bandpass calibration

- Check BP data phase v. frequency also




- Normalise bandpass solutions
- Flux scale may differ or be unset
- In applycal, use interp='nearest' to allow extrapolation
- May need to select timeranges



# Visibility errors and noise

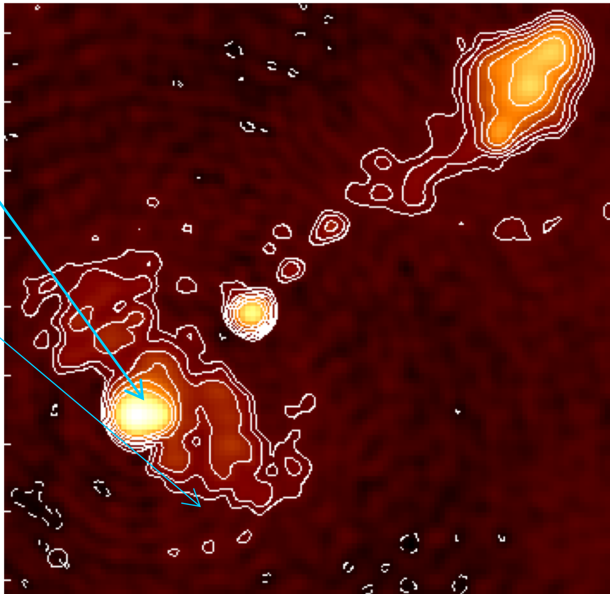
- Lowest possible noise is 'thermal' limit based on $T_{sys}$:

$$\sigma_{sys} = \frac{\langle T_{sys} \rangle}{\eta_A A_{eff} \sqrt{N\,(N-1)/2\ \Delta\nu\ \Delta t N_{pol}}}$$

  - Where $T_{sys} = \frac{1}{\eta_A e^{-\tau_{atm}}} \left[ T_{Rx} + \eta_A T_{sky} + (1-\eta_A) T_{amb} \right]$
- So you can only improve on this by
  - Bigger/more efficient antennas ($A_{eff}$ , $h_A$) or more ($N$)
  - Lower noise Rx and/or $T_{sky}$ (observing conditions)
  - Or, for a given array
  - Observe for longer/wider bandwidth

# What accuracy is needed?

- ➩What is the effect on imaging of visibility errors?
- ➩How good does the calibration need to be?
- ➩How bright is your target?
- ➩Is the peak bright enough to self-cal?
- ➩How faint are the weakest
- ➩features of interest?



# What accuracy is needed?

- Faint target: need to reach thermal noise
- Bright target: may be dynamic range limited
- Need to perfect calibration and imaging
- Astrometry:
- Need high phase accuracy for position accuracy
- Special strategies
- Several phase reference sources
- Can use multiple elevations/frequencies to measure delay and antenna positions with high accuracy

# Phase errors and dynamic range

➪Simplified: flat, linear array, $N$ antennas

☞ Single integration observation of a point source

➪Direction such that we only need to consider $u$ axis

☞ $N(N-1)/2$ visibilities

➪Each baseline visibility is a d spike in the $uv$ plane

☞ All but one are 'perfect' (unit amplitude, zero phase)

➪These have $V(u) = \mathrm{d}(u - u_k)$ for the $k^{\mathrm{th}}$ baseline

➪Phase error on baseline length $u_0$ of $\phi_\varepsilon$ radians

➪$V(u) = \mathrm{d}(u - u_0)\, e^{-i\theta\varepsilon}$

# Phase errors and dynamic range

➪Image is formed by Fourier transform

☞ $I(x) = \int V(u)\, e^{i2pux}\, du$

➪Each baseline contributes at position $u_k$ and complex conjugate $-u_k$ in the visibility plane

➪Evaluating the term in the integral for each of the $[N(N-1)/2]-1$ good baselines gives $2\cos(2pu_k x)$

➪Bad baseline gives $2\cos(2pu_0 x - \phi_\varepsilon)$

☞ $\sim 2[\cos(2pu_0 x) + \phi_\varepsilon \sin(2pu_0 x)]$ for small $\phi_\varepsilon$ (in radians)

➪The image integral thus sums to

$$I(x) = 2\phi_\epsilon \sin(2\pi u_0 x) + 2 \sum_{k=1}^{N(N-1)/2} \cos(2\pi u_k x)$$

# Phase errors and dynamic range

➾The synthesised beam is given by

$$B(x) = 2\sum_{k=1}^{N(N-1)/2} \cos(2\pi u_k x) = N(N\text{-}1) \text{ for } u = 0$$

➾Deconvolution is the subtraction of the beam from the image leaving the residual error

$$R(x) = \left[2\phi_\epsilon \sin(2\pi u_0 x) + 2\sum_{k=1}^{N(N-1)/2} \cos(2\pi u_k x)\right] - 2\sum_{k=1}^{N(N-1)/2} \cos(2\pi u_k x)$$
$$= 2\phi_\epsilon \sin(2\pi u_0 x)$$

➾an 'odd' sinusoidal function of amplitude $2f_e$, period $1/u_0$

➾To maintain the flux scale, integrals are normalised

# Calibration errors and dynamic range

- The rms of the residual

$$R(x) = \frac{2\phi_\epsilon \sin(2\pi u_0 x)}{N(N-1)}$$

- Over the whole map is $\sqrt{2}\ \phi_\varepsilon / N(N-1)$
- For small phase error $\phi_\varepsilon$, large $N$, the ratio of the peak / noise residual is thus
  - Dynamic range $D_B(\phi_\varepsilon) \sim I(x) / R(x) \sim N^2 / \sqrt{2}\ \phi_\varepsilon$
  - e.g., radians (5°)~0.09
- Amplitude error $\varepsilon$ on a single baseline has the effect

$V(u) = (1+\varepsilon)\delta(u - u_0)\, e^{-i\phi}$ leading (via a cos function) to

- Dynamic range $D_B(\varepsilon) \sim N^2 / \sqrt{2}\ \varepsilon$
- **A phase error of 5° is as bad as a 10% amp error**
- **Phase errors are sin (odd), amp are cos (even)**

# Calibration errors and dynamic range

⇨So far considered one-baseline error, one integration

⇨All baselines to one antenna affected by same error: ☞($N$-1) bad baselines (~$N$ for large $N$)

☞$\boldsymbol{D}_{\mathbf{ant}} = D_B/(N-1) = [N^2/(N-1)]/\sqrt{2}\phi_\varepsilon \sim \boldsymbol{N/\sqrt{2}\phi_\varepsilon}$

⇨If all baselines are affected by random noise,

⇨$\boldsymbol{D}_{\mathbf{all}} = D_B/\sqrt{[N(N-1)/2]} = \sqrt{[N(N-1)/2]}/\phi_\varepsilon \sim \boldsymbol{N/\phi_\varepsilon}$

⇨These expressions are valid if errors are correlated in time, e.g. single phase-ref scan, not much change in $u$ (or $v$)

⇨For M periods (scans?) between which noise is uncorrelated

☞ Dynamic range is increased to $\boldsymbol{D}_{\mathbf{all}} \sim \sqrt{\mathbf{M}}\,\boldsymbol{N/\phi_\varepsilon}$

# Calibration for good dynamic range

➩Implications so far: take a 10-antenna array

☞Twelve independent scans on a target, phase reference and other calibration applied, well edited

➩Residual phase scatter 20° : $D_{all} \sim \sqrt{M}\, N/\phi_\varepsilon$

➩~ 100 dynamic range limit

☞Can you improve by self-calibration?

➩No if map noise have reached the $T_{sys}$ limit and remaining errors are pure noise. If not:

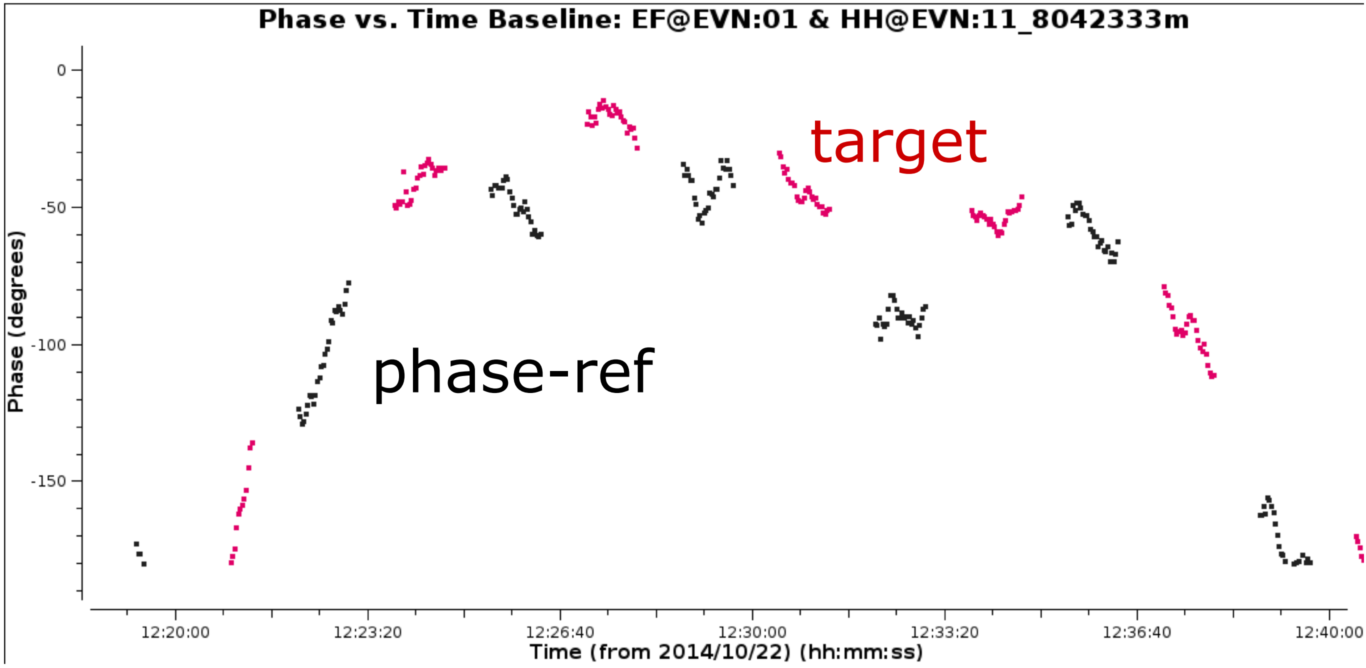
➩Maybe, if some antennas are still imperfectly calibrated

☞Calibrate per antenna, per scan (or longer)

➩Need potential S/N per interval high enough to get $\phi_\varepsilon < 20°$

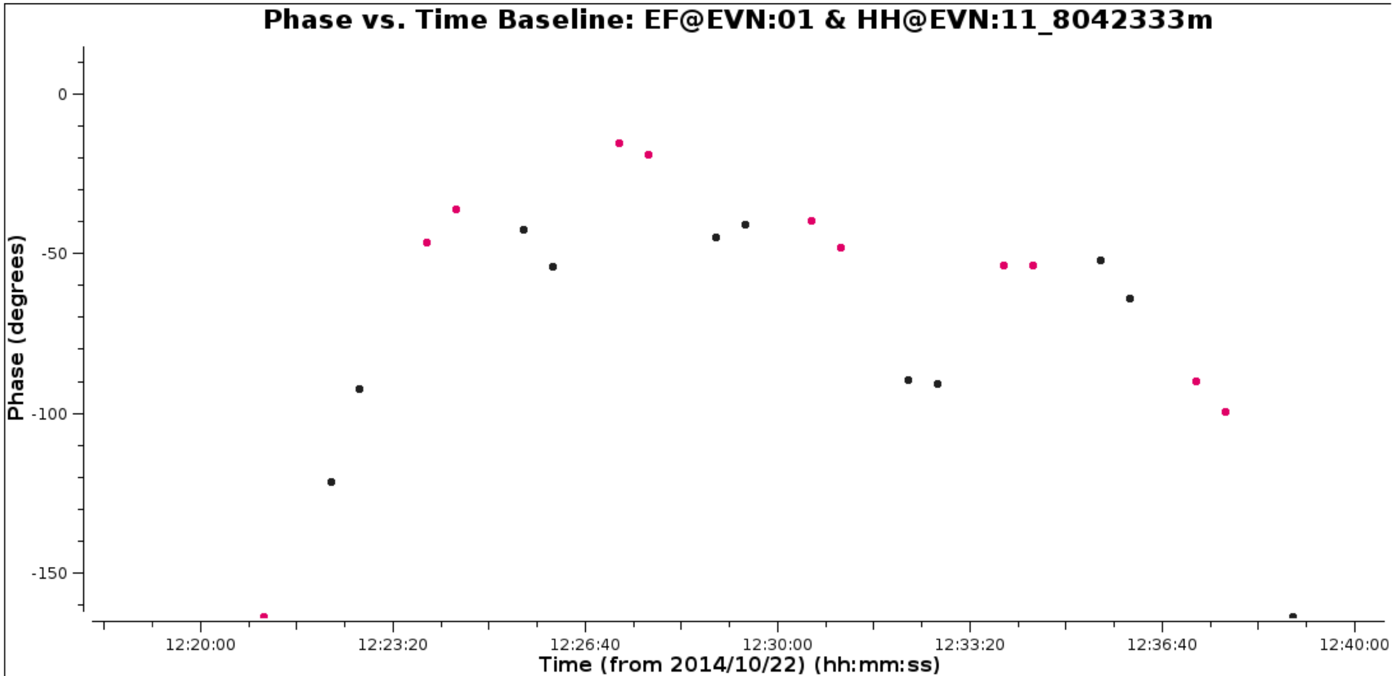
# Time-dependent phase cal

- ➯Apply bandpass/delay corrections
- ➯Phase reference source:
  - ☞ Need to interpolate solutions to target
- ➯Does the phase-ref phase track the target phase?
- ➯Consistent trend seen here
  - ☞Target wiggles may be structure
  - ☞Some deviations



# Time-dependent phase cal

- ➯Need to interpolate phase-ref solutions to target
  - ☞Ideally no more than 2 solutions per phase-ref scan
- ➯Allows simple linear interpolation
  - ☞Must track phase properly
- ➯Check enough S/N in e.g. half scan
  - ☞Seeing low scatter by eye is OK!
- ➯Previous plot with 30-s averaging
- ➯30-s corrections will track phase better than per-scan



# Time-dependent amp cal

➪Apply phase solutions first to allow longer solint for amplitude calibration

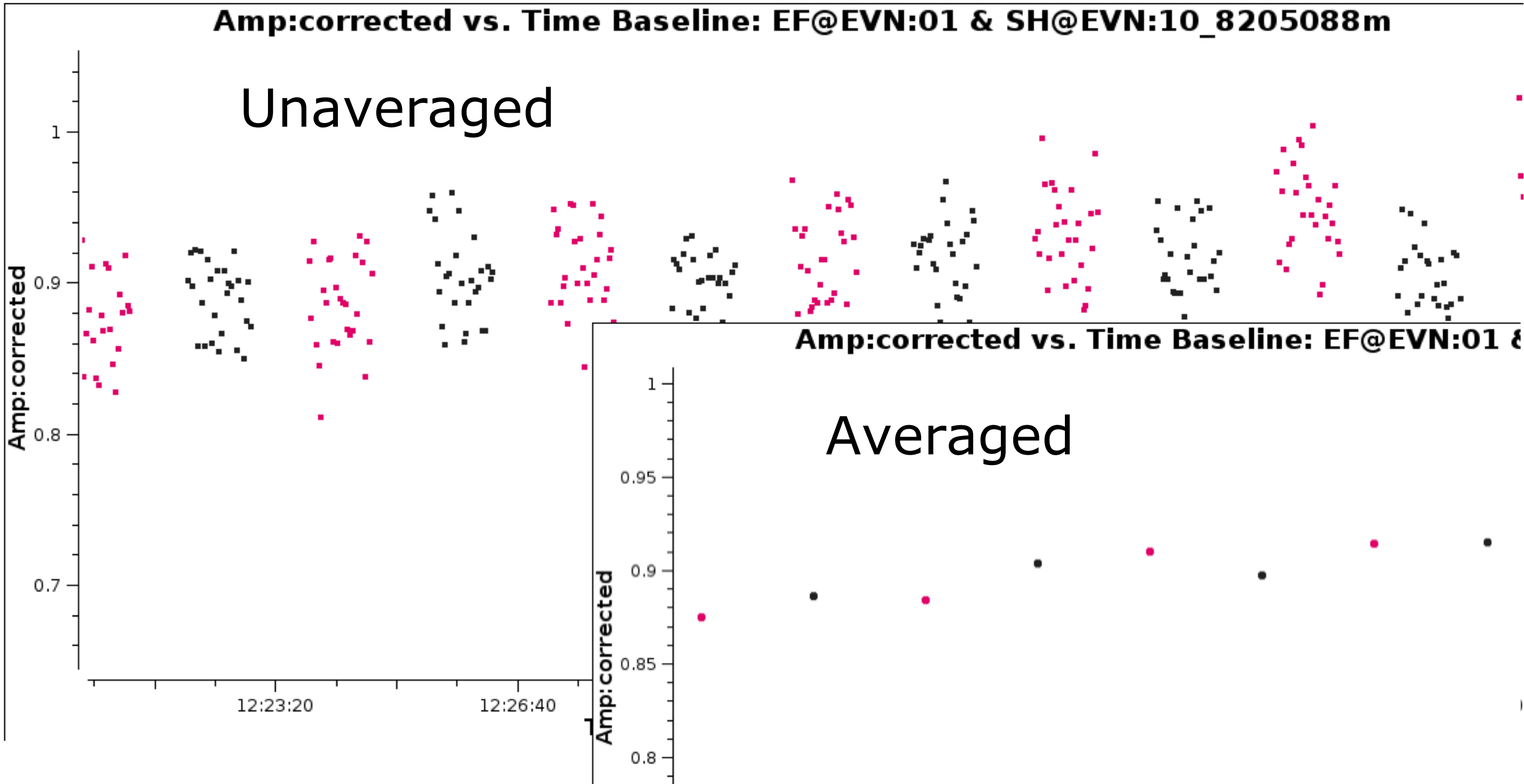
☞Avoid decorrelation

➪If necessary, use shorter phase-only solint just for this

➪Amp scatter per scan usually just noise

☞Average whole scan

☞Solutions will track changes OK

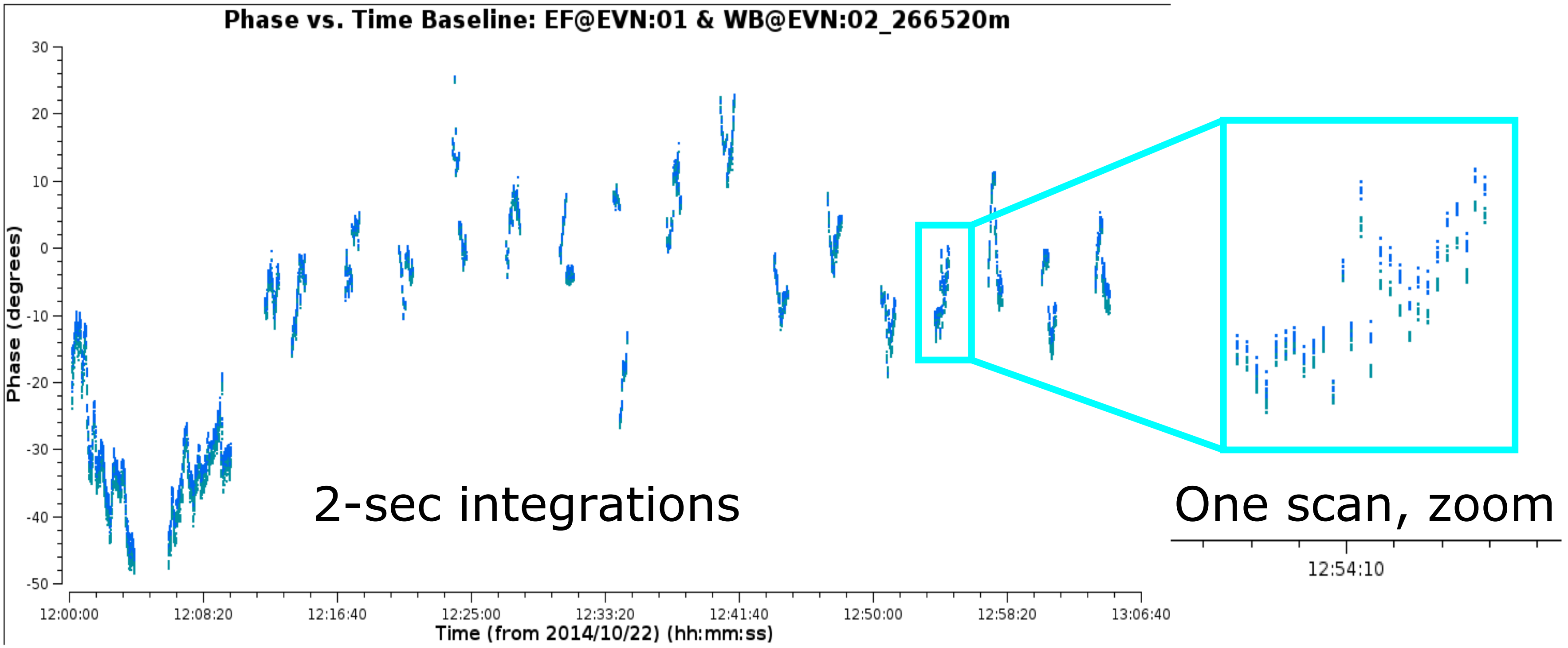


# Self-cal timescales

➪Target phase (after phs-ref corrections) changes rapidly

☞May be partly source structure, but seen even on short b'lines

➪Not just random noise even on 10-sec timescales



# Calibration timescales